Studying latent affective dynamics with a Bayesian state-space approach

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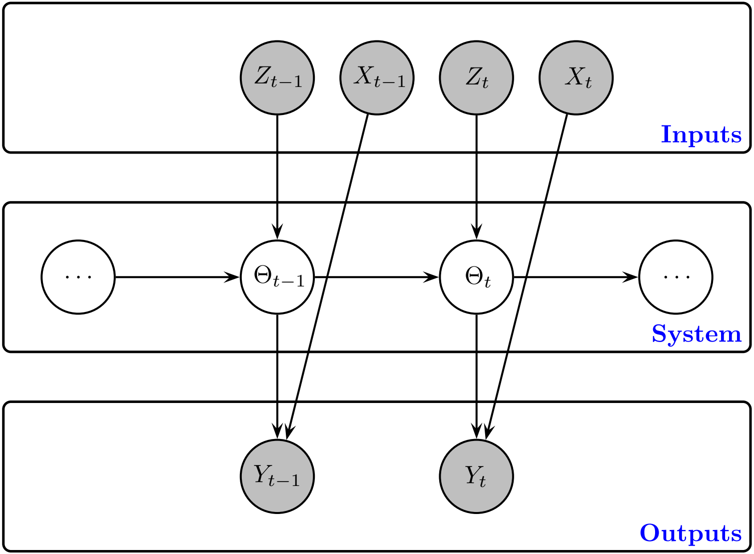
Linear dynamical system theory

- *Inputs (observed):* control / noise / demographic variables
- *System (unobserved):* latent states with dynamical dependencies
- *Outputs (observed):* measurements of interest


Z_{t-1}
X_{t-1}
Z_t
X_t
Inputs
$\cdots$
Θ_{t-1}
Θ_t
$\cdots$
System
Y_{t-1}
Y_t
Outputs

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Model structure

- Transition equation: latent dynamical system
- Observation equation: observed variables

Transition equation

$$Y_t \sim \text{Gaussian}\left(\Psi\Theta_t + \Gamma X_t, \Sigma_\epsilon\right)$$

$$\Theta_t \sim \text{Gaussian}\left(\Phi\Theta_{t-1} + \Delta Z_t, \Sigma_\eta\right)$$

- Θ_t = states
- Θ_{t-1} = states at previous observation moment
- Z_t = state covariates

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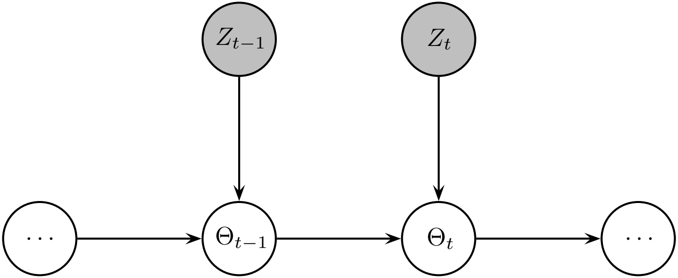
- Θ_t = states
- Θ_{t-1} = states at previous observation moment
- Z_t = state covariates
- Φ = transition matrix (autocorrelations & cross-lagged relations)
- Δ = state covariate regression coefficient matrix
- Σ_η = innovation variance/covariance matrix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Observation equation

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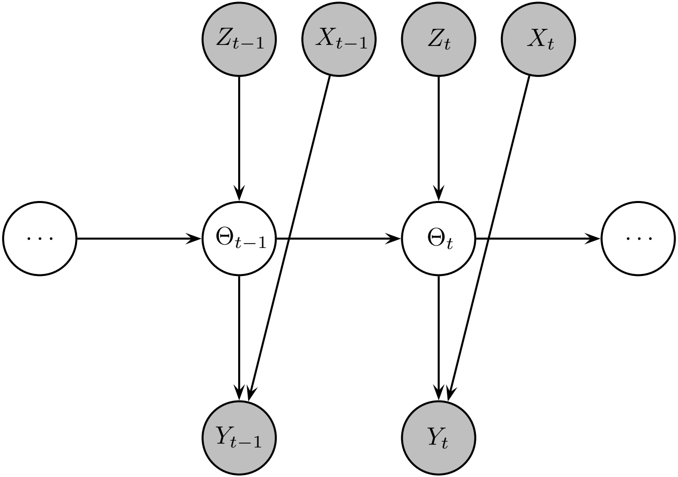
- Y_t = observations
- Θ_t = states
- X_t = observation covariates
- Ψ = design matrix (mapping $\Theta_t \rightarrow Y_t$)
- Γ = observation covariate regression coefficient matrix
- Σ_ϵ = observation error variance/covariance matrix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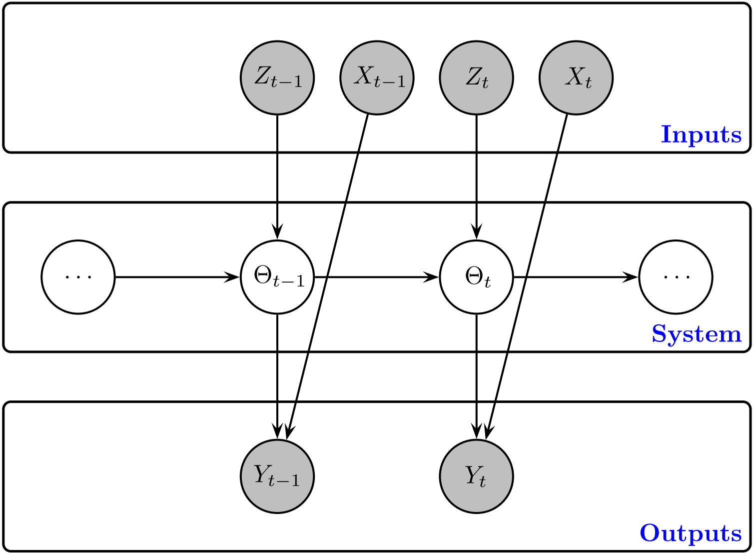
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Regime-switching MLGSS model

$$Y_t \sim \text{Gaussian}\left(\Psi_t \Theta_t + \Gamma_t X_t, \Sigma_{\epsilon t}\right)$$
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$$\begin{pmatrix} \Psi_t, \Gamma_t, \Sigma_{\epsilon t} \\ \Phi_t, \Delta_t, \Sigma_{\eta t} \end{pmatrix} = \begin{cases} \begin{pmatrix} \Psi_1, \Gamma_1, \Sigma_{\epsilon 1} \\ \Phi_1, \Delta_1, \Sigma_{\eta 1} \end{pmatrix} & \text{if } R_t = 1 \\ \quad \cdots & \\ \begin{pmatrix} \Psi_r, \Gamma_r, \Sigma_{\epsilon r} \\ \Phi_r, \Delta_r, \Sigma_{\eta r} \end{pmatrix} & \text{if } R_t = r \end{cases}$$

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Gibbs sampling

- Bayesian estimation method, iterative Monte Carlo simulation
- Each parameter has conditional sampling distribution
- Blocking of parameters efficient for certain models

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Blocked Gibbs sampler

1. Sampling distribution Θ
2. Sampling distribution $\Psi_1, \ldots, \Psi_r, \Gamma_1, \ldots, \Gamma_r, \Sigma_\epsilon$
3. Sampling distribution $\Phi_1, \ldots, \Phi_r, \Delta_1, \ldots, \Delta_r, \Sigma_\eta$

Blocked Gibbs sampler

1. Sampling distribution Θ
   $\longrightarrow$ Forward-filtering backward sampling (Carter & Kohn, 1994,1996)
2. Sampling distribution $\Psi_1, \ldots, \Psi_r, \Gamma_1, \ldots, \Gamma_r, \Sigma_\epsilon$
3. Sampling distribution $\Phi_1, \ldots, \Phi_r, \Delta_1, \ldots, \Delta_r, \Sigma_\eta$

$\Longrightarrow$ **Kalman filter** $\Longrightarrow$

$$\cdots \longrightarrow \Theta_{t-1} \longrightarrow \Theta_t \longrightarrow \cdots$$

$\Rightarrow$ **Kalman filter** $\Rightarrow$

$$\cdots \longrightarrow \Theta_{t-1} \longrightarrow \Theta_t \longrightarrow \cdots$$

$\Leftarrow$ **Backward-sampling** $\Leftarrow$

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   $\longrightarrow$ Method of decomposition (Zellner, 1962)
3. Sampling distribution $\Phi_1, \ldots, \Phi_r, \Delta_1, \ldots, \Delta_r, \Sigma_\eta$
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Parameters observation equation

$\Sigma_{\epsilon} \rightarrow$ Inverse Wishart

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Parameters observation equation

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Parameters transition equation

$\Sigma_\eta \rightarrow$ Inverse Wishart

$\Phi_1, \dots, \Phi_r \mid \Sigma_\eta \rightarrow$ Multivariate Gaussian

$\Gamma_1, \dots, \Gamma_r \mid \Sigma_\eta, \Phi_1, \dots, \Phi_r \rightarrow$ Multivariate Gaussian

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- High: emotions are adapted to the social context
  Low: emotions are insensitive to the context

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Emotional inertia

- High: emotions don't change much over time
  Low: emotions constantly fluctuate over time

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Emotional inertia

- High: emotions don't change much over time
  Low: emotions constantly fluctuate over time
- Autocorrelation of emotional process (ϕ_{em})
  Kuppens, Allen & Sheeber (submitted)

Oregon adolescent interaction data

- 141 adolescents (72 depressed, 69 normal)
- Adolescent, father and mother in laboratory
- Problem solving task (unpleasant)
- One measure/second during 18 minutes ($n = 1080$)

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- Heart rate (HR_t) “Fight-or-flight”
- Respiratory sinus arrhythmia (RSA_t) “Rest-and-digest”

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Measurements

- Baseline RSA (QRSA)
- Heart rate (HR_t) "Fight-or-flight"
- Respiratory sinus arrhythmia (RSA_t) "Rest-and-digest"
- Anger of adolescent ($\mathrm{AdAnger}_t$)

Basic MLGSS Model

- $[Y_{\mathrm{HR}} \;\; Y_{\mathrm{RSA}}]_t \sim \text{Gaussian}\left(I_2[\Theta_{\mathrm{HR}} \;\; \Theta_{\mathrm{RSA}}]_t, \Sigma_\epsilon\right)$

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Correlation(QRSA,$\hat{\phi}_{\text{HR}}$)

	$\hat{\phi}_{\text{HR}}$
QRSA (normal)	-.28
QRSA (depressed)	-.64

Regime-switching MLGSS Model

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  $[\Theta_{\text{HR}} \;\; \Theta_{\text{RSA}}]_t \sim \text{Gaussian}\left(\Phi_t[\Theta_{\text{HR}} \;\; \Theta_{\text{RSA}}]_{t-1}, \Sigma_\eta\right)$

  $$\Phi_t = \begin{cases} \Phi_1 \text{ if AdAnger}_t = 0 \text{ (no anger)} \\ \Phi_2 \text{ if AdAnger}_t = 1 \text{ (anger)} \end{cases}$$

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Correlation(QRSA,$\hat{\phi}_{\text{HR}}$)

	$\hat{\phi}_{1,\text{HR}}$	$\hat{\phi}_{2,\text{HR}}$
QRSA (normal)	-.26	-.10
QRSA (depressed)	-.67	-.65

	$\hat{\phi}_{\text{HR}}$	$\hat{\phi}_{1,\text{HR}}$	$\hat{\phi}_{2,\text{HR}}$
QRSA (normal)	-.28	-.26	-.10
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Results

- Negative relation between QRSA and $\hat{\phi}_{\text{HR}}$:
  The higher (lower) the degree of emotion regulation (QRSA), the weaker (stronger) the emotional inertia ($\hat{\phi}_{\text{HR}}$)

	$\hat{\phi}_{\text{HR}}$	$\hat{\phi}_{1,\text{HR}}$	$\hat{\phi}_{2,\text{HR}}$
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Results

- Negative relation between QRSA and $\hat{\phi}_{\text{HR}}$:
  The higher (lower) the degree of emotion regulation (QRSA), the weaker (stronger) the emotional inertia ($\hat{\phi}_{\text{HR}}$)
- Scarce "angry" data points
- Asymmetry between normal and depressed?

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1. Linear dynamical systems
2. State-space models
3. Bayesian estimation
4. Application
5. **Conclusion**

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1. Does emotional inertia exist for (latent) psychophysiological processes?

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2. Discrete-time state-space models provide interesting framework for studying latent affective dynamics

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2. Discrete-time state-space models provide interesting framework for studying latent affective dynamics
3. Model extensions of interest:
   - Hierarchical
   - Latent regime variable
   - Non-Gaussian errors
   - Model selection

References

- Gyurak, A., & Ayduk, O. (2008). Resting respiratory sinus arrhythmia buffers against rejection sensitivity via emotion control. *Emotion, 8*, pp. 458-467.
- Butler, E. A., Wilhelm, F. H., & Gross, J. J. (2006). Respiratory sinus arrhythmia, emotion, and emotion regulation during social interaction. *Psychophysiology, 43*, pp. 612-622.
- Kuppens, P., Allen, N., & Sheeber, L. (submitted). Emotional inertia in psychological processes.

Questions?